Problem 1

In the following problems, find the limit of the given sequence as $n \to \infty$.

$$\frac{n^2 + 5n^3}{2n^3 + 3\sqrt{4 + n^6}}$$

Solution

Take the limit as $n \to \infty$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{n^2 + 5n^3}{2n^3 + 3\sqrt{4 + n^6}} &= \lim_{n\to\infty} \frac{n^2 + 5n^3}{2n^3 + 3\sqrt{n^6\left(\frac{4}{n^6} + 1\right)}} \\
&= \lim_{n\to\infty} \frac{n^2 + 5n^3}{2n^3 + 3n^3\sqrt{\frac{4}{n^6} + 1}} \\
&= \lim_{n\to\infty} \frac{\frac{1}{n} + 5}{2 + 3\sqrt{\frac{4}{n^6} + 1}} \\
&= \frac{0 + 5}{2 + 3\sqrt{0 + 1}} \\
&= 1
\end{aligned}$$